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A Quantitative Methodology for Estimating Total System Cost Risk

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This report presents a complete, detailed methodology for deriving quantitative estimates of program/system cost-risk. The procedures are completely algebraic and direct. They do not rely on Monte Carlo or Method of Moments techniques. Principle sources of uncertainty that contribute to cost-risk are identified for each of the four costing approaches treated in the methodology:

1. Parametric Costing Approach (Cost = $a + bx^c$)

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

2. Engineering (Bottom-Up/Grass Roots) Costing Approach

3. Analogous (Scaling) Costing Approach

4. Constant Multiplier Costing Approach

Methods for generating estimates of the initial sources of uncertainty are described and illustrated. The initial sources of uncertainty are processed algebraically through the mechanics of the costing approaches to yield estimates of the uncertainty (cost-risk) in the costs of the individual system/WBS elements. The report then examines the conventional statistical methodology for aggregating uncertainties - the root-sum-square technique - and shows why this technique is inapplicable. An alternative, recommended, methodology is described and illustrated. At this point in the methodology, a system/program cost along with a measure of the cost uncertainty is available. The final part of the methodology describes and illustrates how a cost-probability relationship can be developed from the generated data. Included in the discussion are some guidelines for selecting a discrete risk-adjusted cost estimate (R-ACE). The report concludes with a discussion of some additional considerations that bear on the practical application of the methodology and a brief mention of cost estimating deficiencies which constitute areas for future research.

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A

SUMMARY

The over-all methodology presented for determining total system cost risk is judged to constitute a significant advance in the area of cost risk determination. Although it admittedly favors pragmatic results over "noncost-effective" statistical rigor, it is basically sound. The fundamental value of the methodology is that it recognizes explicitly and derives analytically impacts on cost of the uncertainties prevailing in both system element estimates and in cost estimating relationships - uncertainties that were formerly ignored, but which contribute significantly to cost estimating "error." Methodologies are presented for arriving at the uncertainty surrounding system element costs that are derived by alternative costing approaches (e.g., parametric, engineering, analogy, and constant multiplier).

It is shown that the uncertainties in the costs of the system elements cannot be validly aggregated by the conventional technique of adding the variances (root-sum-squaring). The proposed approach - which is recognized as being conservative in that the errors are not diminished by the process of error aggregation - consists of adding the standard deviations associated with the costs of the system elements. Pragmatically, the estimate of the total cost error tends to be "reasonable." By generating a cost-probability curve (truncated), the cost risk involved can be perceived. However, for budgetary purposes, a risk-adjusted cost estimate (R-ACE), corresponding to a probability level commensurate with the novelty of the system, needs to be selected. The risk-adjusted cost thus selected should be used for the determination of budgetary requirements. A management reserve base on the difference between the R-ACE and the expected cost (0.5 probability) should be established inasmuch as the expected cost is virtually certain to be exceeded.

By compiling data on the results of the application of this methodology, particularly on the resultant accuracy of the selected R-ACE values, specific guidelines or relationships for selecting probabilities commensurate with the novelty of systems can be established. Complex, first generation systems might necessitate a R-ACE at the 0.9 probability level, whereas fifth generation systems might be accurately estimated at the 0.7 probability level.

Identification of such probability-determining guidelines could further improve the accuracy of system cost estimating.

The presentation of this methodology is but the first step. Its application together with documentation of results, will permit the identification and correction of any deficiencies. Toward this end, the methodology developed for propagating error (uncertainty) in the parametric costing approach together with the methodology for aggregating the cost uncertainty have been incorporated into the Air Force Space Division computerized version of the Unmanned Spacecraft Cost Model.*

The potential applicability of this methodology or its elements extends beyond cost risk estimation per se. The methodology for the identification of errors, their propagation through mathematical relationships and ultimate aggregation appears to be applicable to technical fields other than cost estimating.

By continuing identification, reduction, and eventually elimination of sources of cost estimating error, more accurate cost estimates will be achieved. The methodology developed and described herein is a step in that direction. Consistently accurate system cost estimates, however, should realistically be viewed as an illusive goal because of the inevitable presence of inherently random or uncontrollable events that impact costs. (Even with complete estimating accuracy at all steps, a 0.9 probability level cost will be exceeded 10% of the time because of random influences.) Nevertheless, significant, further improvements can be achieved through better understanding of the sources of cost errors and the decisions that influence the procurement of the system being costed.

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* Jose Gutierrez of The Aerospace Corporation was responsible for modifying the cost model (on the Hewlett-Packard 9845) such that both numerical results and cost-probability plots would be generated.

PREFACE

The research culminating in this report began several years ago as a personal effort. The first product - a briefing titled A Proposed Generic Approach to Conceptual Phase Risk Analysis - was presented at the Space Systems Cost Analysis Group (SSCAG) meeting at Sunnyvale, Calif. on 23 September 1981. It described general error propagation techniques and error aggregation methodologies and problems. The next product was a briefing on a significant achievement in error propagation which was accomplished with the aid of Dr. R. H. Huddlestone. This briefing, in the form of an actual costing example, displayed the use of the algebraic technique developed for propagating uncertainty in an independent variable, through the uncertainty inherent in a cost estimating relationship (CER) based on the independent variable, so as to generate an estimate of the uncertainty in the derived cost. The briefing, titled Example of the Deterministic Method for Generating Cost-Probability Relationships was presented at the Denver SSCAG meeting on 20 January 1982. The application of this error propagation methodology was then documented and presented on 21 May 1982 at the SSCAG meeting in Stevenage, England, in a paper titled An Algebraic Technique for Estimating the Uncertainty of a Parametrically Derived Cost Estimate.

Support of this effort under Air Force contract F04701-81-C-0082 commenced in August 1982. The focus of the contracted effort was threefold: (1) to develop a methodology for aggregating the uncertainties in the cost elements so as to arrive at an estimate of the uncertainty in the total system cost; (2) to incorporate both the parametric CER uncertainty estimating methodology and the uncertainty aggregation methodology into the Air Force Space Division's computerized (HP 9845) cost model; and (3) to document the methodologies developed (this report). Upon outlining the report, a conspicuous deficiency became apparent. Although methodologies were presented for estimating the uncertainties in system costs derived by parametric, engineering (bottom-up/grass roots), and factor approaches, a methodology for estimating the uncertainty in a cost derived by the analogy approach was lacking. A personal effort (nonfunded because it was beyond the initial scope of the contracted effort) was initiated to develop this methodology.

A significant breakthrough in this effort was achieved on 3 January 1983, again with the aid of Dr. Huddlestone. A briefing on cost uncertainty determination methodologies under the four costing approaches along with the cost aggregation methodology developed was presented at the Los Angeles SSCAG meeting on 13 January 1983 under the title Presentation of Methodologies Developed for Estimating Total System Cost Risk for Four Alternative Costing Techniques.

This report documents the methodologies developed for generating quantitative estimates of total system cost risk. It includes detailed descriptions and examples of techniques for (1) estimating the uncertainty in costs derived by the four basic cost estimating approaches: parametric, engineering (bottom-up/grass roots), analogy, and constant multiplier; (2) aggregating the derived cost uncertainties; (3) generating cost-risk (probability) relationships; and (4) selecting a R-ACE value. Because cost estimates and analyses are frequently generated by individuals with non-mathematical/engineering backgrounds, descriptions of the methodology as well as the illustrative examples were written with these individuals in mind.

During the past several years, a number of SSCAG members contributed - through challenging, informal discussions of aspects of the problem - to the development of the methodologies and views presented. Specifically, the support of Col. Lowell Maxwell (USAF-Ret.), who was chairman of SSCAG when these efforts commenced should be recognized, as well as that of his successor as SSCAG chairman, Col. Dan Fitzgerald, under whose support this effort was concluded. In addition, the members of the SSCAG Risk Analysis Task Team - Robert Black, Edward (Ned) Dodson, Alvin Owens, Robert Seldon and James Wilder - should be acknowledged for their stimulating and penetrating comments during the evolution of the methodology.

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I. INTRODUCTION

A. BACKGROUND

Virtually all cost estimates - whether for private, commercial, or governmental systems - are presented as fixed point estimates without qualification, and are commonly assumed to possess a high level of certainty. However, even a cursory examination of the nature of costs reveals that they are intrinsically highly variable. Unless the cost of a commodity or system element is artificially fixed, it will vary locally, regionally, nationally and, to compound the variability, temporally. Raw material costs, labor costs, utility costs all vary, if not locally, then regionally and nationally.* Complex systems are essentially the results of labor (managerial, engineering, manufacturing, etc.), equipment, and energy applied to raw materials. If the costs of these elements possess an inherent variability, then the resulting aggregated cost of the systems created would likewise display a corresponding variability. The foregoing sources of cost uncertainty are further compounded by uncertainties inherent in both the specific details of the system being costed, and in the amounts of labor and materials that will be required.

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* According to the 1980 Dodge Manual, published by McGraw Hill, the wage rates for laborers in Raleigh, N.C. was $6.06 per hour, whereas 430 air miles away in Cleveland, Ohio, it was $17.79 per hour or 190% greater. Salaries of professional engineers likewise show considerable variability as reported in Professional Engineer Income and Salary - 1981, published by the National Society of Professional Engineers. Based on samples of hundreds of professional engineers, the difference in median salary between engineers in Columbus and Detroit (a distance of less than 200 miles) was more than 20%. A 20% median salary differential was also shown between hundreds of professional engineers in Houston and San Antonio, approximately 200 miles apart. In the summer of 1980, the residents of Atlanta paid 4.25 cents per kilowatt-hour of electricity, whereas New Yorkers paid 11.77 cents per kilowatt-hour, or 177% more. During the same period, the rate per kilowatt-hour in Indianapolis was 4.96 cents while 175 miles away in Columbus it was 7.77 cents or 57% more. (Based on a survey by the National Association of Regulatory Utility Commissioners). Material costs depend on extraction and processing labor costs, processing energy costs, and transportation costs, all of which are variable geographically and temporally.

Regardless of what technique may be used to generate system cost estimates, the variability inherent in the cost of the system elements, when combined with the intrinsic uncertainty in the cost-impacting details of the system together with such factors as inflation impacts, should make any unqualified point cost estimate highly suspect. A cost estimate at best is only an approximate representation of the expected system cost rather than the precise price that it is too commonly assumed to be.

B. PURPOSE/OBJECTIVE

This report presents quantitative methodologies for identifying the cost variability within the individual system elements, a methodology for aggregating the estimated uncertainties in the costs of the system elements, and a methodology for depicting the system cost uncertainty by a range of system costs with their associated probabilities of occurrence. Thus, the inherent uncertainties prevailing in a system and in its costs would be reflected by a cost-probability relationship rather than as an ambiguous point cost estimate.

Two basic techniques are currently used for generating the desired cost-probability relationships: the Monte Carlo approach and the Method of Moments approach.[1,2,3] Both of these methods, however, are of such complexity that the use of a computer (or at least a programmable calculator) is essential. Approximately 500 computer iterations are required by the Monte Carlo method in order to generate a relatively smooth cost-probability relationship. The need for a simple, straightforward technique for generating a quantitative estimate of the cost uncertainty (risk) has long been recognized. The objective of this report is to describe, in detail, the direct, quantitative methodologies developed for deriving subsystem cost uncertainties, aggregating these uncertainties into a total system cost uncertainty, and finally deriving a probability-related cost estimate.

II. CONTENTS AND SCOPE

The subsequent methodology for determining total system cost risk consists of three distinct sequential steps or areas. The first area focuses on methods for estimating the uncertainty associated with the cost of individual system elements. Methods are developed and illustrated for determining these cost uncertainties when the cost estimates are derived by any of the four basic costing approaches: parametric, engineering (bottom-up/grass roots), analogy, or constant multiplier (factor). The major sources of uncertainty that are ultimately manifested in cost uncertainty are treated within each costing approach. These sources of cost uncertainty are identified in Table 1 for each of the four costing approaches. In the parametric costing approach, the two major sources of uncertainty considered are the uncertainty in the independent parameter (e.g., subsystem weight*) and the uncertainty in the cost estimating relationship (CER). In the engineering approach, the uncertainty in the man-hours (or quantity of material required) is considered as well as the uncertainty in the corresponding wage rate (or cost per unit quantity of material). In the analogous or scaling approach, the uncertainty in the system parameter (e.g., weight, power) and the uncertainty in the scaling exponent are considered. In the constant multiplier or factor approach, the only source of uncertainty is in the estimate of the system parameter which is multiplied by a fixed factor to obtain the estimated cost.

Factors contributing to cost uncertainty, such as technological developments required, funding stretch-out or schedule slippage, and design changes are not specifically considered. However, they are intrinsically incorporated in a normalized manner within the historic data points on which the parametric CERs are based and also in a similar manner, in the known system cost (C_2), and within the exponent in the analogous system costing approach.

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* Weight really is a dependent variable, but because cost is often highly correlated with it, it is frequently used in CERs as an expedient surrogate for a normalized mix of independent system parameters, which are also highly correlated with it.

Table 1. Major Sources of Cost Uncertainty for Four Basic Costing Approaches

	System Element Costing Approach	Equation	Sources of Uncertainty
1.	Parametric	Cost = $a + bP^c$	- System parameter P - Cost model (std. error of est.)
2.	Engineering	Cost = (MH) x (Rate) Cost = (Mat) x (Price)	- Man-hours required - Rate (skill level) - Material required (quantity) - Price (per unit quantity)
3.	Analogy	$\frac{Cost_1}{Cost_2} = \left(\frac{P_1}{P_2}\right)^Y$ or $Cost_1 = Cost_2 \left(\frac{P_1}{P_2}\right)^Y$	- System parameter P_1 - Scaling exponent Y
4.	Factor	Cost = P x K	- System parameter P

The second area in the overall methodology development presents a procedure for aggregating the individual system element cost uncertainties that were derived by one of the four approaches. The result of the aggregation is a composite uncertainty that reflects the uncertainty (risk) associated with the total system cost. Application of the aggregation methodology developed is illustrated by an example.

The third area presents a methodology for developing a cost versus probability relationship. It also includes some guidelines for deriving a meaningful, specific, R-ACE from the cost-probability relationship for program funding purposes. The methodologies developed are then summarized, and future efforts for enhancing cost estimating methodologies are identified.

III. METHODOLOGIES FOR ESTIMATING COST UNCERTAINTY WITHIN SYSTEM ELEMENTS

The methodologies for estimating the cost uncertainty within the individual system elements under the four approaches shown in Table 1 depend on estimates of the mean $\bar{x}$ and standard deviation σ of the cost-driving parameters. The following section describes two techniques for generating values for the mean and standard deviations from engineering estimates of the cost-driving parameters. The two are the beta distribution and the triangular distribution techniques.

A. TECHNIQUES FOR GENERATING QUANTITATIVE ESTIMATES OF COST-DRIVING VARIABLES AND THEIR UNCERTAINTIES

1. BETA DISTRIBUTION TECHNIQUE

This technique traces its origin to PERT (Program Evaluation and Review Technique) where it was widely used to generate time estimates for events in a scheduling network. In the beta distribution technique as used here, three estimates of a system cost-driving parameter are solicited from the knowledgeable engineering specialist most knowledgeable about that system: low, most likely, and high. The low, or optimistic, estimate should correspond approximately to a value that would be realized only under the most fortuitous circumstances - a subjective probability somewhere in the 0.01 to 0.10 range. The most likely estimate is just that - the mode. The high, or pessimistic, estimate should correspond to a value that reflects the ultimate working of Murphy's Law - a subjective probability in the 0.99 to 0.90 range. Thus, if the three values are:

a = low estimate

m = most likely estimate

b = high estimate

then the mean value $\bar{x}$ can be estimated by

$$\bar{x} = \frac{1}{6}(a + 4m + b) \tag{1}$$

and the standard deviation about the mean can be estimated by:

$$\sigma_{\bar{x}} = \frac{1}{6}(b - a) \tag{2}$$

2. TRIANGULAR DISTRIBUTION TECHNIQUE

The triangular distribution technique is generally preferred over the more common beta distribution technique.[4] In the triangular technique, the identical estimates - low (a), most likely (m), and high (b) - estimates of the cost-driving parameter (e.g., weight) are obtained from the knowledgeable system/subsystem specialist. The mean $\bar{x}$ can then be estimated by

$$\bar{x} = \frac{1}{3}(a + m + b) \tag{3}$$

and the standard deviation σ by

$$\sigma = \sqrt{\frac{1}{18}\left[(b - a)^2 + (m - a)(m - b)\right]} \tag{4}$$

Generally, subsystem characteristics, performance requirements or design constraints enable the knowledgeable subsystem specialist to readily arrive at the low and high estimates. The most likely estimate usually presents the greatest difficulty because of a significant degree of uncertainty (indifferance) in the broad mid-range area. The nature of the computations for $\bar{x}$ and σ, however, tends to be forgiving and reduces the percentage of error in estimating m. Figure 1 shows the general uncertainty or indifference in providing an estimate of the most likely value, m.

A measure of the sensitivity of $\bar{x}$ and σ to the selection of m may be seen from an analysis of the three triangular distributions shown in Figure 2. It assumes that curve A is selected when the actual correct values are depicted by curves B or C. Table 2 shows the sensitivity of the calculated values of $\bar{x}$ and σ with respect to the assumption of m.

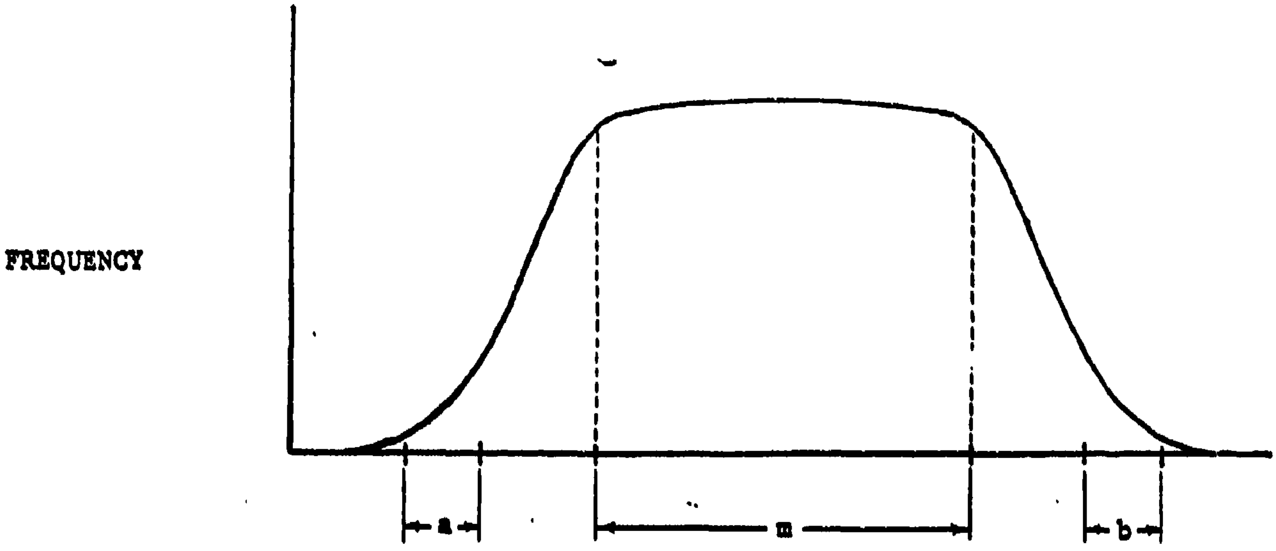

Figure 1. Schematic of General Uncertainty in Estimating the Most Likely Value

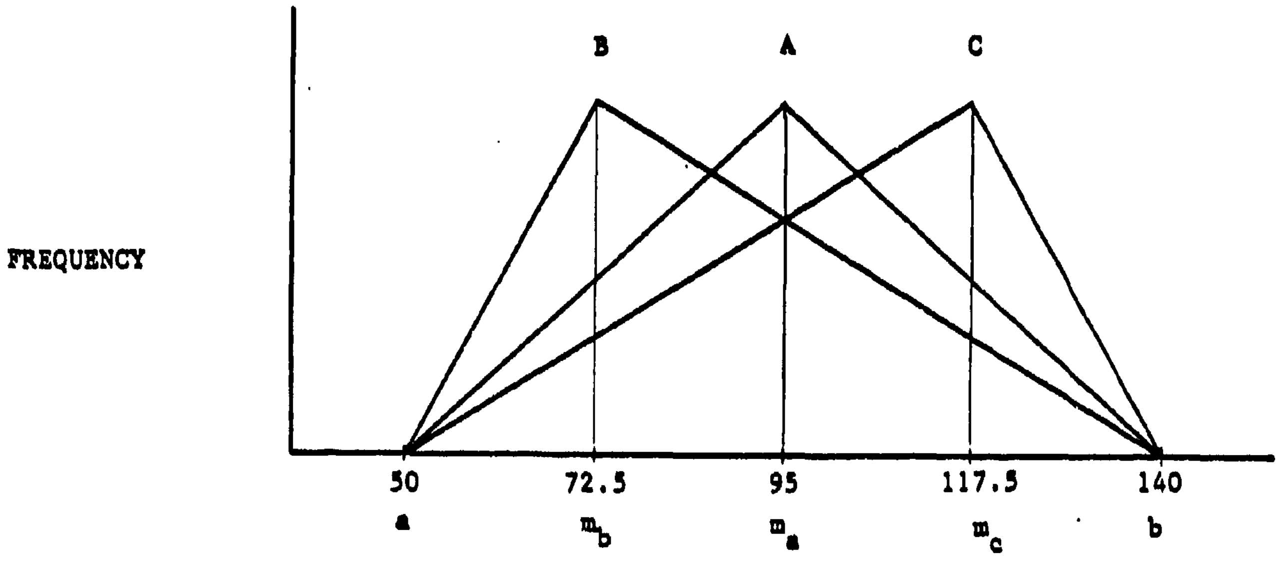

Figure 2. Assumption of Most Likely Values at 25%, 50%, and 75% of Range

Table 2. Sensitivity of Calculated $\bar{x}$ and σ with Respect to Assumption of m

Curve	a	m	Δm	b	$\bar{x}$	$\Delta\bar{x}$	σ	$\Delta\sigma$
A	50	95	0	140	95	0	18.4	0
B	50	72.5	+ 31.0%	140	87.5	+ 8.6%	19.1	3.7%
C	50	117.5	- 19.1%	140	102.5	- 7.3%	19.1	3.7%

The refinement of intentionally incorporating substantial skewness into the distribution by the estimate of the most likely value does not appear to be warranted. However, if there are recognizable and justifiable reasons for estimating the most likely value at other than approximately the mid-range point, this certainly should be done.

B. PARAMETRIC COSTING APPROACH

This section describes and illustrates an algebraic technique for estimating the uncertainty within a parametrically derived cost estimate.

1. PROBLEM

Parametrically derived cost estimates generally have two major sources of uncertainty: the uncertainty inherent in the parametric descriptor of the system (e.g., weight), and the uncertainty associated with the cost-estimating regression equation, which uses the parametric descriptor to derive the cost estimate. The interrelationship between these two basic sources of uncertainty is shown in Figure 3.

The essence of the problem is how to propagate the uncertainty in the independent parameter that is used to predict the cost, through the uncertainty in the cost estimating equation, so as to obtain an estimate of the resultant composite uncertainty in the derived cost estimate.

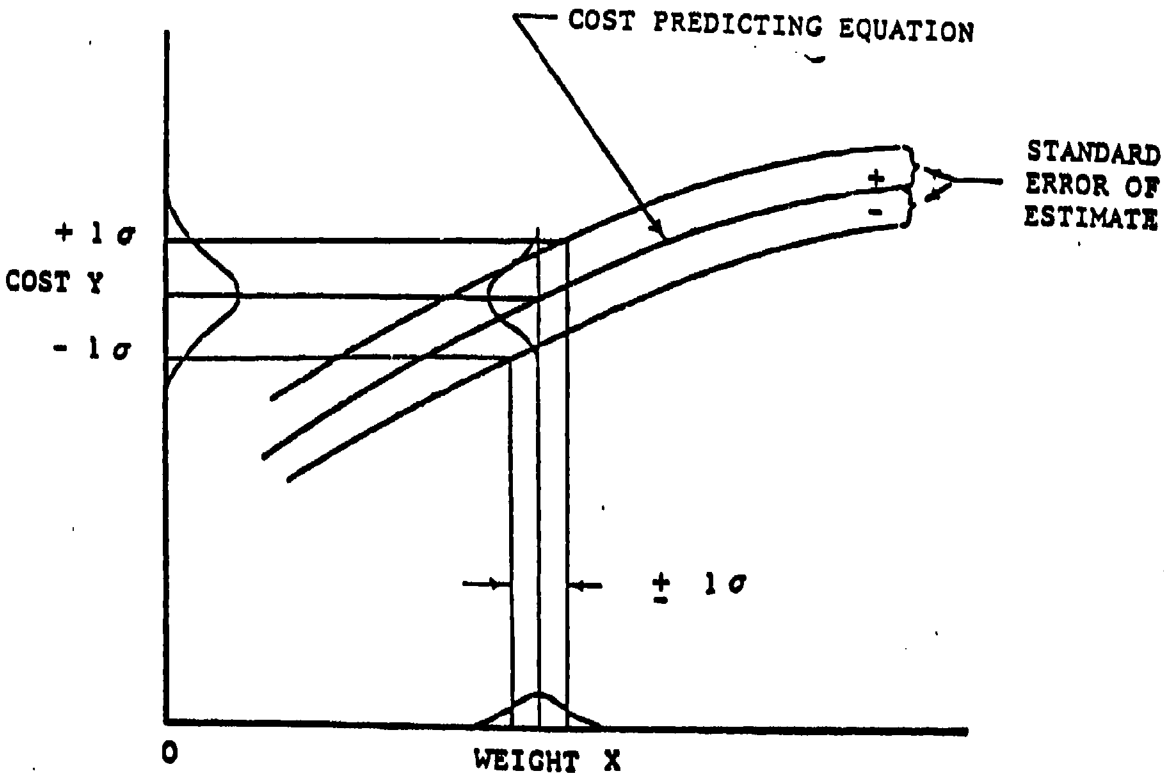

Figure 3. Schematic Illustration of the Uncertainty Propagation Problem

2. ASSUMPTIONS/REQUIREMENTS

The following assumptions are implicit in the proposed methodology:

a. An estimate of the uncertainty in the independent variable (e.g., weight) can be generated (usually by beta or triangular distribution techniques such as described in the preceding section).

b. A measure of the uncertainty in the cost-predictive regression equation is obtainable in terms of the standard error of estimate.[5,6,7,8]

c. The uncertainty distributions associated with both the independent variable and the regression equation are symmetrical. (Neither the Monte Carlo approach nor the Method of Moments approach is restricted to this assumption.)

d. The independent variable is within the predictive range (bounds) of the regression equation. Extrapolation beyond the bounds of the regression equation could introduce significant additional sources of uncertainty that are not treated here.

e. The cost-predictive equation is of the general form:

$$Y = a + bx^c$$

For example,

$$Y = a + bx^c$$
$$Y = bx^c \quad \text{(a, the Y intercept is equal to zero)}$$
$$Y = a + bx \quad \text{(c is equal to 1)}$$
$$Y = bx \quad \text{(a is equal to zero, and c is equal to 1)}$$

3. METHODOLOGY FOR DETERMINING THE UNCERTAINTY IN A PARAMETRICALLY DERIVED COST ESTIMATE

The application of the algebraic relationship requires the following data:

a. For the independent variable (e.g., system weight), an estimate of the mean value $\bar{x}$, and an estimate of the standard deviation σ_x.

b. A cost predicting regression equation (of the general form $Y = a + bx^c$) together with an estimate of the standard error (SE) about the regression curve.

Knowing both the mean and standard deviation of the independent variable (e.g., weight), as well as the regression line equation with its standard error of estimate. Then the standard deviation of the dependent variable (e.g., cost) σ_Y can be estimated by the relationship:*

$$\sigma_Y^2 = SE^2 + \frac{b^2}{4}\left[(\bar{x} + \sigma_x)^c - (\bar{x} - \sigma_x)^c\right]^2 \tag{5}$$

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* R. H. Huddlestone, Prediction Error Statistics for a Nonlinear Cost Risk Model, The Aerospace Corporation, Interoffice Correspondence (13 January 1982). (Not available for public distribution).

It should be noted that this relationship is independent of a (the Y intercept) and so will hold true for regression equations of the form $Y = bx^c$. In the linear case $Y = a + bx$, it reduces to a standard result. The application of this methodology is illustrated by the following example.

4. APPLICATION OF PARAMETRIC COST UNCERTAINTY DETERMINATION

The following example is based on the Telemetry, Tracking and Command (TT&C) subsystem data for generating first unit cost and was extracted from the Unmanned Spacecraft Cost Model, SD-TR-81-45, dated June 1981 which was prepared by Air Force Space Division/ACC. The TT&C subsystem design weight of 81.6 lb found on page VII-3 of the subject document was modified by the data for triangular distribution parameters shown in Table D-1, page D-4, as follows so as to represent typical data that might be encountered in an actual case:

Basic design weight: 81.6 lb	
Low estimate (- 50%)	40.8 lb = a
Mode or most likely estimate (+ 2.9%)	84.0 lb = m
High estimate (+ 58%)	128.9 lb = b

The mean value, assuming a triangular distribution, can be derived by Eq. (3), so that the mean weight in this case is

$$\bar{x} = 1/3\ (40.8 + 84.0 + 128.9) = 84.6 \text{ lb}$$

Likewise, the standard deviation for a triangular distribution can be estimated by Eq. (4), which in this case is

$$\sigma_x = \sqrt{1/18\left[(128.9 - 40.8)^2 + (84 - 40.8)(84 - 128.9)\right]} = 18.0 \text{ lb.}$$

The first unit cost-predicting regression equation for TT&C is given on page IV-11 of the referenced document to be

$$Y = 42.43 + 35.93x^{.93}$$

where Y = First unit cost in thousands of FY 79 dollars and
x = TT&C weight in pounds

By substituting the mean weight of 84.6 lb for x in this equation and solving, an estimated first unit cost of $Y = 42.43 + 35.93(84.6)^{.93} =$ \$2270.4 is obtained. The standard error of estimate (SE) for this equation is reported as 713.9. By inserting the corresponding values into the cost uncertainty estimating equation (5), the standard deviation for the TT&C first unit cost is found to be

$$\sigma_Y^2 = 713.9^2 + \frac{35.93^2}{4}\left[(84.6 + 18.0)^{.93} - (84.6 - 18.0)^{.93}\right]^2$$

or

$$\sigma_Y = \$839.2$$

The contribution of the uncertainty in the weight (σ_x = 18.0 lb) resulted in an increase in uncertainty of

$$\frac{839.2 - 713.9}{713.9} \times 100 = 17.6\%$$

over that attributable to the standard error of estimate alone.

C. ENGINEERING (BOTTOM-UP/GRASS ROOTS) COSTING APPROACH

The engineering approach is probably the most widely used technique for preparing cost estimates of systems that are in the latter stages of development.

1. PROBLEM

Engineering cost estimates focus primarily on labor and material costs and secondarily on energy and processing costs. Cost estimates of system elements are usually derived by multiplying the quantity estimated to be required times a unit rate (e.g., man-hours times dollars per hour; pounds times dollars per pound, units times dollars per unit). In this case, the two major sources of uncertainty are the uncertainty in the estimate of the required quantity (e.g., man-hours, pounds), and the uncertainty in the estimate of the cost per unit (e.g., dollars per man-hour, dollars per pound). The essence of this problem is how to combine the uncertainty in the quantity required with the uncertainty in the dollar rate per unit quantity so as to obtain an estimate of the uncertainty in the product cost.

2. ASSUMPTIONS/REQUIREMENTS

The following are required for the implementation of this uncertainty propagating methodology:

a. An estimate of the quantity required a along with a measure of the uncertainty in this estimate in terms of the standard deviation σ_a. The estimated standard deviation can be derived by either beta or triangular distribution techniques as previously described.

b. An estimate of the rate per unit quantity b together with an estimate of its standard deviation σ_b. This too can be derived by beta or triangular distribution techniques as previously mentioned.

c. The uncertainty distributions associated with both the quantity estimate and the rate per unit quantity are symmetrical.

d. The quantity and rate per unit quantity are assumed to be independent.

3. METHODOLOGY FOR DETERMINING THE UNCERTAINTY IN AN ENGINEERING (BOTTOM-UP/GRASS ROOTS) COST ESTIMATE

As stated in the assumptions, in order to apply this methodology, it is necessary to have estimates of the quantity (mean) a and its standard deviation σ_a, as well as the rate per unit quantity (mean) b and its standard deviation σ_b. These values are used to calculate the coefficient of variation (called the "fractional standard deviation" by some authors) for both the quantity and the rate. The coefficient of variation S is the standard deviation divided by the mean. Thus,

Coefficient of variation for quantity: $S_a = \frac{\sigma_a}{a}$ (6)

Coefficient of variation for rate: $S_b = \frac{\sigma_b}{b}$

The cost is, then, the product (P$) that results from multiplying the quantity a times the rate per unit quantity b

$$P\$ = a \times b$$

The standard deviation σ_P about the product P$ is then found by:

$$\sigma_P = ab\sqrt{S_a^2 + S_b^2} \quad (7)$$

4. APPLICATION OF COST UNCERTAINTY DETERMINATION IN THE ENGINEERING APPROACH

If 5000 diverse engineering man-hours a were expected to be required to develop a specific element with a standard deviation σ_a of 500 man-hours, and the mean burdened rate per engineering man-hour b was estimated to be $60 per hour with a standard deviation σ_b of $10 per hour, then the estimated cost of developing the element is

$$P\$ = a \times b$$
$$P\$ = 5000 \times 60$$
$$P\$ = 300{,}000$$

The coefficients of variation are

$$\text{Quantity:} \quad S_a = \frac{\sigma_a}{a} = \frac{500}{5000} = 0.10$$

$$\text{Rate:} \quad S_b = \frac{\sigma_b}{b} = \frac{10}{60} = 0.167$$

and the standard deviation for the product of a times b is

$$\sigma_P = ab\sqrt{S_a^2 + S_b^2}$$

$$= 5000 \times 60\sqrt{0.10^2 + 0.167^2}$$

$$\sigma_P = \$58{,}400$$

Whereas, the individual coefficients of variation were 0.10 and 0.167, the coefficient of variation in the resulting product is

$$\frac{\sigma_P}{P\$} = \frac{58,400}{300,000} = 0.195$$

or greater than that of either of the factors.

D. ANALOGOUS (SCALING) COSTING APPROACH

The analogous or scaling costing approach is used in cases where sufficient historical data are lacking so that meaningful/valid CERs cannot be developed by statistical (regression) techniques, but where there is a close similarity between the system (subsystem) being costed and an existing system (subsystem) whose cost is known. This approach is widely used in the chemical process industry to generate preliminary cost estimates for new chemical plants or equipment based on the known cost of a pilot plant or smaller scale installation. It is commonly referred to in the literature as the "six-tenths factor" costing approach even though scaling factors other than 0.6 are often used.

1. PROBLEM

In the analogous costing approach, a system similar, or as the term implies - analogous - to the one being costed exists. Key characteristics (e.g., weight, output), proven by experience to be highly correlated with cost, are known for both systems - the analogous system and the system being costed. Thus, the variables in the cost estimating equation are

C_N = cost of the new system (sought)
C_O = known, actual cost of old, analogous, existing system
P_N = cost-correlated characteristic of new system
P_O = known, cost-correlated characteristic of old, analogous, existing system
K = experientially based scaling exponent

and the cost estimating relationship is

$$\frac{\text{Cost of New System}}{\text{Cost of Old System}} = \left(\frac{\text{Characteristic of New System}}{\text{Characteristic of Old System}}\right)^{\text{Exponent}}$$

Thus,

$$\frac{C_N}{C_0} = \left(\frac{P_N}{P_0}\right)^K \tag{8}$$

or

$$C_N = C_0\left(\frac{P_N}{P_0}\right)^K$$

The characteristic P_0 and cost C_0 of the analogous, existing system are precisely known (no uncertainty). The two sources of input uncertainty in this approach are associated with the characteristic of the new system being costed P_N, and with the uncertainty in the scaling exponent K.

A perusal of exponents used to cost chemical industry elements reveals exponents ranging from approximately 0.1 to over 3.0, although most fall between 0.4 and 0.9.[9] Scaling exponents for fighter or transport aircraft subsystems generally range from 0.70 to 1.0, with most falling between 0.8 and 0.9.[10]

2. ASSUMPTIONS/REQUIREMENTS

The following assumptions and requirements underlie the application of the costing by analogy approach:

a. An estimate of the mean value of the cost-correlated characteristic P_N (e.g., weight, power output) of the new system or subsystem is required along with an estimate of the uncertainty about the mean value in terms of the standard deviation σ_{PN}.

b. An expected value of the scaling exponent K is required together with its uncertainty in terms of the standard deviation σ_K.

c. The uncertainty distributions associated with the characteristic P_N and with the exponent K are both assumed to be symmetrical.

3. METHODOLOGY FOR DETERMINING THE UNCERTAINTY IN A COST ESTIMATE DERIVED BY THE ANALOGY (SCALING) APPROACH

This methodology, developed by Dr. R. H. Huddlestone*, differs from those previously presented in that the uncertainty in the cost is not uniquely determined by a specific equation, but rather by a procedure as follows: The two key sources of uncertainty in the analogy method are (1) the estimate of the new system's characteristic, and (2) the estimate of the scaling exponent. Estimates of these, along with their standard deviations as measures of their uncertainty, need to be obtained, which can be done by the beta or triangular distribution techniques as described in Sections III A 1 and III A 2.

The standard deviations are then applied to the mean values of P_N and K so as to obtain + 1 and - 1 standard deviation values for each of the two parameters - four points in all.

$$P_N + 1\sigma_{PN}$$
$$P_N - 1\sigma_{PN}$$
$$K + 1\sigma_K$$
$$K - 1\sigma_K$$

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* R. H. Huddlestone, Estimated Error in Costing by Analogy, The Aerospace Corporation, Interoffice Correspondence (7 January 1983). (Not available for public distribution.)

These values are then used to determine four estimates of the cost C_N, where

$$C_N = C_O\left(\frac{P_N}{P_O}\right)^K$$

The four values of C_N are obtained by substituting the following four sets of values into the preceding equation and solving for C_N:

1. $P_N - 1\sigma_{PN}$, $K - 1\sigma_K$
2. $P_N - 1\sigma_{PN}$, $K + 1\sigma_K$
3. $P_N + 1\sigma_{PN}$, $K - 1\sigma_K$
4. $P_N + 1\sigma_{PN}$, $K + 1\sigma_K$

The mean of the four C_N values is then obtained, and the standard deviation of the four values of C_N from the mean C_N is generated. The best estimate of the cost of the new system is the mean value, and the standard deviation thus calculated is the sought estimate of the uncertainty in the system cost.

4. APPLICATION OF COST UNCERTAINTY DETERMINATION IN THE ANALOGY APPROACH

A system analogous to the one being costed was known to have weighed 200 lb (P_O) and cost \$10 million (C_O). The new system is expected to weigh 800 lb (P_N) with a standard deviation of 200 lb (σ_{PN}). The expected scaling exponent K_E is estimated to be 0.6 with a standard deviation σ_K of 0.2. Thus,

C_O = \$10 M
P_O = 200 lb
P_{NL} (low, $- 1\sigma_{PN}$) = 600 lb
P_{NE} (expected) = 800 lb, σ_{PN} = 200
P_{NH} (high, $+ 1\sigma_{PN}$) = 1000 lb
and K_L (low, $- 1\sigma_K$) = 0.4
K_E (expected) = 0.6, σ_K = 0.2
K_H (high, $+ 1\sigma_K$) = 0.8

By using the equation

$$C_N = C_O\left(\frac{P_N}{P_O}\right)^K$$

and the four preceding sets of values for P_N and K, four estimates of C_N are obtained. The best estimate of C_N can be shown to lie within the bounds of these four points, as illustrated in Figure 4. An estimate of the standard deviation σ_{CN} can be obtained by determining the standard deviation of the value of C_N for these four points from the mean C_N, as shown in Table 3.

Table 3. Illustrative Data and Computational Results for Determining Estimated Cost and Cost Uncertainty by the Analogy Approach

Point	P_N		K		Calc. C_N	Dev. (D) From Mean C_N	D^2
1.	Low	600	Low	0.4	15.52	8.20	67.24
2.	Low	600	High	0.8	24.08	0.36	0.13
3.	High	1000	Low	0.4	19.04	4.68	21.90
4.	High	1000	High	0.8	36.24	12.52	156.75
			Total C_N =		94.88	Total D^2 =	246.02
			Mean C_N =		23.72		

$$\text{Then, } \sigma_{CN} = \sqrt{\frac{\Sigma D^2}{N}} = \sqrt{\frac{246.02}{4}} = 7.84$$

Thus, the estimated cost of the new system C_N is \$23.7 million with a standard deviation σ_{CN} of \$7.8 million.

E. CONSTANT MULTIPLIER (FACTOR) COSTING APPROACH

In some instances, the cost of subsystems can be estimated by multiplying the cost estimate of a related system element by a fixed factor(s). The factor might reflect a spares cost factor, inflation factor, learning curve factor, or some other fixed parameter.

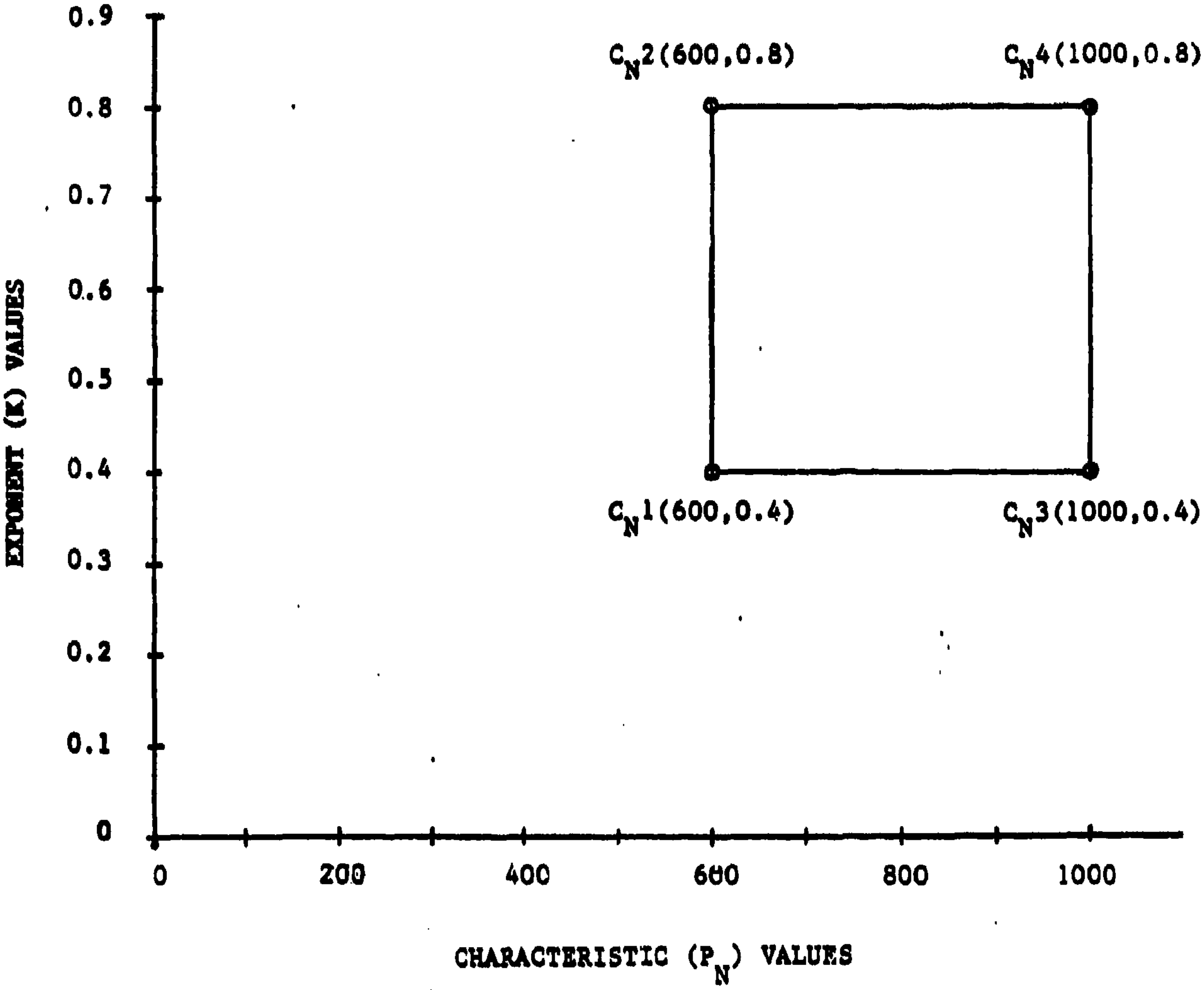

Figure 4. Schematic of Bounds of System Cost C_N

1. PROBLEM

In this approach, the factor is fixed - by edict or mathematical principles. The cost, which is multiplied by the factor, however, does possess uncertainty - either actual historical variability or developed estimated cost uncertainty. The problem in this approach is to ascertain the variability in a cost developed by multiplying a cost estimate, that has some level of uncertainty by a fixed constant.

2. ASSUMPTIONS/REQUIREMENTS

The following are required for the implementation of the constant multiplier (factor) costing approach:

a. An estimate of the historical or previously generated (expected) cost of the subsystem/element, along with a measure of the uncertainty about the expected cost as expressed by the standard deviation.

b. The uncertainty distribution about the expected cost is symmetrical.

3. METHODOLOGY FOR DETERMINING THE UNCERTAINTY IN A COST ESTIMATE GENERATED BY A CONSTANT MULTIPLIER

The methodology for propagating the uncertainty in a variable when it is multiplied by a constant is well established,[11,12] and is presented here for completeness. The methodology consists of multiplying the standard deviation of the original, independent variable by the constant in order to obtain the standard deviation in the resulting, dependent variable. Thus, if the initial cost is C, its standard deviation σ, and the constant multiplier K, then the new cost C_2 and its standard deviation σ_2 are found by:

$$C_2 = KC_1 \tag{9}$$

and

$$\sigma_2 = K\sigma_1 \tag{10}$$

4. APPLICATION OF COST UNCERTAINTY DETERMINATION IN THE CONSTANT MULTIPLIER APPROACH

If a system was estimated to cost \$850 million in some past-year's dollars with a standard deviation of \$200 million, and the inflation factor to bring the past costs to current-year dollars is 1.3, then the new (current-year) cost and its standard deviation are obtained as follows:

K = constant (e.g., inflation factor)

C_1 = cost in past-year dollars

C_2 = cost in inflated current-year dollars

σ_1 = uncertainty associated with the cost expressed in past-year dollars

σ_2 = uncertainty associated with adjusted-for-inflation cost

$$C_2 = KC_1$$
$$C_2 = 1.3 \times \$850 \text{ million}$$
$$C_2 = \$1105 \text{ million}$$

and

$$\sigma_2 = K\sigma_1$$
$$\sigma_2 = 1.3 \times \$200 \text{ million}$$
$$\sigma_2 = \$260 \text{ million}$$

Thus, the cost in current-year dollars is found to be \$1105 million, with a standard deviation of \$260 million.

IV. METHODOLOGY FOR AGGREGATING THE COST ELEMENT UNCERTAINTIES

In the preceding section, methodologies were presented for developing cost uncertainties when individual system elements are costed by one of the four basic costing approaches. The costs of the individual system elements are obviously aggregated by adding, but it is not clear how the uncertainties (e.g., σs) associated with the system element costs should be combined to reflect the uncertainty in the aggregated cost. This section identifies a methodology, along with its rationale, for aggregating the cost uncertainties.

A. FALLACY IN APPLICATION OF CLASSICAL/CONVENTIONAL STATISTICS

Conventional statistics holds that if variables are additive, then their variances (the square of the standard deviation or σ^2) are also additive. Thus, the variance about the sum of a series of variables, each with its own variance, is found by adding the variances as follows:

	Variable	Standard Deviation	Variance
	a	σ_a	σ_a^2
	b	σ_b	σ_b^2
	c	σ_c	σ_c^2
	d	σ_d	σ_d^2
Sum	S		

The variance about the sum σ_S^2 then, is

$$\sigma_S^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2 + \sigma_d^2$$

The standard deviation about the sum σ_s may then be derived by taking the square root of both sides of the equation; thus,

$$\sigma_S = \sqrt{\sigma_a^2 + \sigma_b^2 + \sigma_c^2 + \sigma_d^2}$$

This is the conventional root-sum-square (RSS) relationship.* What happens, however, when this relationship is applied to system cost data?

Suppose a system is composed of three independent elements, each with a cost of 50 and a standard deviation about the cost equal to 10% of the cost, or 5. The total system cost and its standard deviation would be derived thusly:

Element	Cost	Standard Deviation
1	50	5_1
2	50	5_2
3	50	5_3
	Total cost = 150	

The standard deviation about the total cost computed by applying the RSS approach would be

$$\sigma_s = \sqrt{5_1^2 + 5_2^2 + 5_3^2} = 8.66$$

The standard deviation, which initially amounted to 10% of the cost of each element is, by the application of the RSS technique, reduced to 5.8% of the sum.

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* A refinement of the RSS technique focuses on the independence or non-independence of the variables being aggregated. If they are indeed all independent, then the conventional, straightforward RSS equation applies. However, if they are nonindependent (correlated), then an additional complexity arises - covariance term(s) need to be added to the sums of the variances.**

** R. H. Huddlestone, Estimated Error in Total System Cost, The Aerospace Corporation, Interoffice Correspondence (30 November 1982) discusses this issue. (Not available for public distribution.)

Now, if the system were twice as large and consisted of six rather than three like elements, each of which again had a standard deviation equal to 10% of the mean, thus,

Element	Cost	Standard Deviation
1	50	5_1
2	50	5_2
3	50	5_3
4	50	5_4
5	50	5_5
6	50	5_6
	Total Cost = 300	

then, the standard deviation about the total cost using the RSS approach would be

$$\sigma_s = \sqrt{5_1^2 + 5_2^2 + 5_3^2 + 5_4^2 + 5_5^2 + 5_6^2}$$

or 12.25. This is but 4.1% of the sum.

If the system were truly gigantic and complex, such that it consisted of 24 statistically independent elements under the above assumptions, then the total cost would be 1200, and the standard deviation using the root-sum-square approach would be 24.49, or now only 2.0% of the total cost! These examples, which show the effect of applying conventional RSS techniques, are summarized in Table 4 below.

Table 4. Potential Impact of Using RSS Technique for Cost Uncertainty Aggregation

No. of Elements	Total Cost	Uncertainty σ	Uncertainty σ as % of Total Cost
3	150	8.66	5.8
6	300	12.25	4.1
24	1200	24.49	2.0

These data show that as the system becomes larger, (e.g., more expensive and complex) the percent uncertainty tends to decrease! This is contrary to all costing experience. Something is wrong. An examination of what is actually occurring will reveal the source of this fallacy.

B. SOURCE OF INAPPLICABILITY OF CONVENTIONAL STATISTICS

1. NATURE OF COMPETITIVE PROCUREMENT PROCESS

When proposals for the development and/or production of a system are received from several qualified contractors, cost is invariably a criterion for selecting the winner. Not necessarily the lowest cost - but rather a reasonable, credible cost. This obviously puts pressure on the competitive bidders to develop low but realistic bids. However, in most system procurements, there is a significant element of uncertainty. This is reflected by the inclusion of prototype development and tests into the program. The tests are in essence demonstrations to ascertain whether the manifold uncertainties pervading the system have been successfully resolved. Tests are expensive. How many tests will be required reflects just one aspect of contractor optimism. Some level of optimism (confidence in the engineering staff's technical competence) must be displayed. A bid based on pessimistic outcomes would surely lose to a more success-oriented competitor. On the other hand, the compulsion to submit a low bid based on substantial optimism would be prudently balanced by experientially derived knowledge of the existence of a potentially malevolent reality (Murphy's Law!). Consequently, modest reserves for a limited number of adverse events are usually incorporated into the bid. Thus, the nature of the competitive procurement process results in bids that (1) tend to be somewhat low, based on both an optimistic view of the contractor's technical competence and prevailing competitive forces, and (2) belong to a two-tailed distribution inasmuch as the cost could be lower if the program were so successful that the reserves were not needed; or higher, if the amount of reserves were inadequate. Invariably, the uncertainties surrounding the costs of all of the individual elements that are aggregated to produce the final bid are two-tailed, even though some tails may be highly skewed. Thus, in aggregating the costs and their uncertainties, a root-sum-square approach would appear valid. The high-tailed cost uncertainties would be offset by the low-tailed cost uncertainties. The population would tend towards normality, and the Central Limit Theorem would hold.

2. MANAGEMENT PRINCIPLES/DECISION DRIVERS

Once a contract is awarded, what was up to that point a "normal" distribution becomes a truncated (one-sided) distribution, and the root-sum-square approach to aggregating the uncertainties is no longer valid. That portion of the cost distribution which lies below the mean or expected cost disappears, so there are no offsetting values for the high portion of the distribution. The reason for this is based on program management motivators or decision drivers. The program manager is acutely aware that the systems being procured have a higher intrinsic value than what is reflected in their cost (or else their acquisition would be uneconomic and imprudent). Both his and his company's reputation depend on the reliable performance of the developed system over its expected life - but in the case of space systems - with stringent weight and/or volume constraints. The anticipated reliability is substantiated by failure mode analyses which pinpoint weak links. Engineers responsible for elements of the space system invariably identify changes that would enhance reliability and improve the system, but usually at an increase in cost, weight, and/or volume. Thus, if the cost is underrunning, a prudent manager would opt to use the available funds to eliminate the ever-present weakest link and so enhance the system's reliability, life, or other key attributes.* It is virtually inconceivable that a program manager would decide to come in below the contract cost while weak links exist that could jeopardize the total success of the system. (How much testing and reliability <u>is</u> enough?)

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* Major General Jasper Welch, USAF, Assistant Deputy Chief of Staff for Research, Development and Acquisition, writing in the December 1982 issue of <u>Electronic Business</u> (pages 55, 56) admonished contractors for merely meeting rather than substantially exceeding negotiated MTBF specifications.

Furthermore, the level of contractor optimism evoked by a competitive procurement process operates to reduce the probability of such a highly successful program that a cost underrun would result. (Some sole-source procurements without cost-depressing competitive pressures have resulted in underruns, but this is deemed to be due to the submittal of a more realistically high bid than would normally be the case in a competitive procurement.) The pressures on the program manager personally and on the contractor in general to deliver the "best" possible system, coupled with the ubiquitous weak links, preclude cost underruns. Consequently, the system costs incurred are not governed by, nor display, randomness about the contract awarded amount. The RSS technique in which variances are summed, assumes a randomizing nature at work, or an "averaging" of the high side values with those below the expected (i.e., mean) value. This does not occur here. There are no "low" values. If the costs of the elements were normally distributed so that some were higher than expected while others were lower (thus producing a bell shaped histogram or curve), then the conventional RSS approach might apply, but this is not the case. The awarded amount - usually the bid price (slightly modified during contract negotiations) - which was two-tailed in its development, becomes a floor once the award is made. The two-tailed, chance element that underlies the RSS technique no longer prevails. Root-sum-squaring should not be used. It generates wrong estimates of the standard deviation because "averaging" does not occur.

The foregoing assessment of the program manager applies equally down the organizational line to the subsystem project engineers and subcontractors. Once a cost goal has been set or subcontract award made (based again on a measure of optimism), that then becomes the cost floor. The best subsystem possible will be developed for the designated cost. However, any adversity encountered, beyond that anticipated in preparation of the proposed bid, will likely result in cost overruns. Thus, the bid/award cost will not be underrun. It can only be met or overrun. This negates the applicability of the root-sum-square technique for aggregating system cost uncertainties.

C. RECOMMENDED APPROACH

The independent parameters (e.g., subsystem weight, CER standard error of estimate) on which cost estimates are based possess an inherent two-tailed probability distribution ($\pm \sigma$). The beta and triangular distribution techniques described provide methods for estimating a two-tailed uncertainty about a mean or expected value. The methodologies described for propagating the uncertainties in the independent variables within the four costing approaches result in the development of two-tailed uncertainty distributions about each element or subsystem cost derived. Now, the purpose of aggregating the element/subsystem costs and their uncertainties is to arrive at a total, composite cost along with its uncertainty that will reflect actuality - what really may occur. Conventionally, aggregation of two-tailed uncertainties is validly done by means of the RSS technique - RSS alone if the variables whose uncertainties are being aggregated are independent, RSS plus a covariance factor if the variables are nonindependent. But, as described in the preceding section, forces prevail that eliminate the lower tail, thus negating the applicability of a RSS approach. The one-sidedness of the cost incurrence is the basis for the recommended approach. Specifically, in aggregating the uncertainties associated with one-sided cost elements, the uncertainties as reflected in the standard deviation associated with each of the aggregated cost elements should be added to arrive at a standard deviation for the aggregated cost.*

D. APPLICATION OF AGGREGATION METHODOLOGY

The following example is based on an actual space system concept. High, low, and most likely weight estimates were obtained for each subsystem element from the respective subsystem specialists. The triangular distribution methodology (discussed and demonstrated in Section III A 2) was used to generate the expected value (mean) and estimated uncertainty (i.e., standard deviation, σ) for each of the subsystem elements. Parametric cost models

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* This conclusion is supported by analyses performed by Dr. R. H. Huddlestone and presented in Estimated Error in Total System Cost, The Aerospace Corporation, Interoffice Correspondence (30 November 1982). (Not available for public distribution.)

were used to determine the subsystem element costs. The standard deviations of the weight estimates were combined with the standard error of estimate of the CERs by means of the parametric error propagation equation (5), thereby providing an estimate of the standard deviation (uncertainty) in the system element cost. Tables 5 and 6 show the subsystem costs by cost element along with the standard deviation for the non-recurring costs and recurring costs, respectively. The coefficients of variation or fractional standard deviations are also shown as a percentage. The standard deviations are aggregated by summation into a total system standard deviation. The standard deviations that would be obtained if RSS techniques were used are shown for comparison.

The total system acquisition cost, not including launch or operations and support costs, is shown in Table 7, along with the aggregated cost uncertainty as reflected by the standard deviation.

Table 5. Aggregation of System Non-Recurring Costs and Their Uncertainties

Non-Recurring Cost in FY 82 Dollars

System Element	Parametric Cost Est. M$	Estimated Std. Dev. σ M$	Coefficient of Variation %
Mission Equip. (M.E.)			
Element A	50.0	3.0	6.0
Element B	225.4	33.2	14.7
Element C	36.3	2.7	7.4
M.E. Subtotal	311.7	38.9 (33.4 RSS)	12.5 (10.7% RSS)
Spacecraft (S/C)			
Structure & Mech.	12.2	2.1	17.2
Thermal Control	3.6	2.1	58.3
Electric Power	14.9	2.9	19.5
Trk., Tel. & Comm'd	53.1	2.6	4.9
Stab. & Control	19.0	7.8	41.1
Aux. Propulsion	3.0	0.9	30.0
S/C Subtotal	105.8	18.4 (9.3 RSS)	17.4 (8.8% RSS)
Integration	15.6	1.9	12.2
Qual. & Space Proto.	472.1	53.4	11.3
Ground Support Equip.	9.6	1.0	10.4
Fee	64.0	8.0	12.5
Total Non-Recurring	978.8	121.6 (64.2 RSS)	12.4 (6.6% RSS)

Table 6. Aggregation of System Recurring Costs and Their Uncertainties

Recurring Costs (buy of six) in FY 1982 Dollars

System Element	Parametric Cost Est. M$	Estimated Std. Dev. σ M$	Coefficient of Variation %
Mission Equip. (M.E.)			
Element A	36.3	3.8	10.5
Element B	269.4	28.4	10.5
Element C	63.6	6.4	10.1
M.E. Subtotal	369.3	38.6 (29.4 RSS)	10.5 (8.0% RSS)
Spacecraft (S/C)			
Structure & Mech.	19.2	3.6	18.8
Thermal Control	4.8	3.4	70.8
Electric Power	66.6	8.3	12.5
Trk., Tel. & Comm'd	95.4	6.3	6.6
Stab. & Control	34.8	7.2	20.7
Aux. Propulsion	6.6	0.3	4.5
S/C Subtotal	227.4	29.1 (13.6 RSS)	12.8 (6.0% RSS)
Integration	18.6	1.9	10.2
Launch Site Support	8.5	1.1	12.9
Fee	43.7	4.9	11.2
Total Recurring	667.5	75.6 (32.8 RSS)	11.3 (4.9% RSS)

Table 7. System Acquisition Cost and Cost Uncertainty (σ)
(In FY 82 Dollars)

Procurement Phase	Estimated Cost M $	Estimated Std. Dev. σ M $	Coefficient of Variation %
Non-Recurring	978.8	121.6	12.4
Recurring (6)	667.5	75.6	11.3
Total	1646.3	197.2	12.0

V. COST-PROBABILITY (COST RISK) RELATIONSHIP

The objective of system cost risk analysis is to portray the financial resources required to acquire a given system in a concise yet realistic manner. A point cost estimate, although concise, does not realistically represent the actual uncertainties that prevail. A cost-probability curve for the given system can depict the measure of cost risk associated with the acquisition of the system.

A. METHODOLOGY FOR GENERATING A COST-PROBABILITY CURVE

The expected (mean) total system cost and its standard deviation - whose generation was described and illustrated in the previous section - are sufficient for the development of a cost-probability curve. The expected cost corresponds to a 0.5 probability. If a normal probability distribution is assumed, reference to cumulative probability tables shows that the expected cost, + 1 standard deviation, corresponds to a 0.841 cumulative probability. This means that the probability is 0.841 or 84.1% that the cost will be less than the amount corresponding to the expected cost, + 1 standard deviation. The expected cost plus twice the standard deviation corresponds to a cumulative probability of 0.977 or 97.7%. Similarly, the expected cost, - 1 standard deviation, corresponds to a 0.159 probability and the expected cost, - 2 standard deviations corresponds to a 0.023 probability. The cost probability curve can be generated by plotting the above points for cost versus probability on standard probability paper and connecting them with a straight line.

The three prerequisites for generating a cost-probability curve are

1. The availability of the expected (mean) cost with its standard deviation.

2. The assumption of normality (for at least the upper tail of the distribution, as discussed in Section C below).

3. The following table of cumulative probabilities versus standard deviations (σ):

	Cumulative Probability	%
Expected Cost - 2σ	0.0228	2.28
Expected Cost - 1σ	0.1587	15.87
Expected Cost (mean)	0.5000	50.00
Expected Cost + 1σ	0.8413	84.13
Expected Cost + 2σ	0.9772	97.72

B. EXAMPLE OF COST-PROBABILITY RELATIONSHIP DETERMINATION

A cost-probability relationship will be developed for the total system cost $1646.3 million, and standard deviation $197.2 million developed in Section IV D and shown in Table 7. The cost probability curve can be developed by plotting the following cost versus percent probability values on probability paper, as illustrated in Figure 5:

	Cost	% Probability
Expected Cost	1646.3	50.0
Expected Cost + 1σ: 1646.3 + 197.2 =	1843.5	84.1
Expected Cost - 1σ: 1646.3 - 197.2 =	1449.1	15.9
Expected Cost + 2σ: 1646.3 + 2(197.2) =	2040.7	97.7
Expected Cost - 2σ: 1646.3 - 2(197.2) =	1251.9	2.3

The steeper the slope of the curve (i.e., relatively narrow cost range about the expected cost), the less the cost risk; the shallower the slope of the curve (i.e., relatively wide/broad cost range about the expected cost), the greater the cost risk.

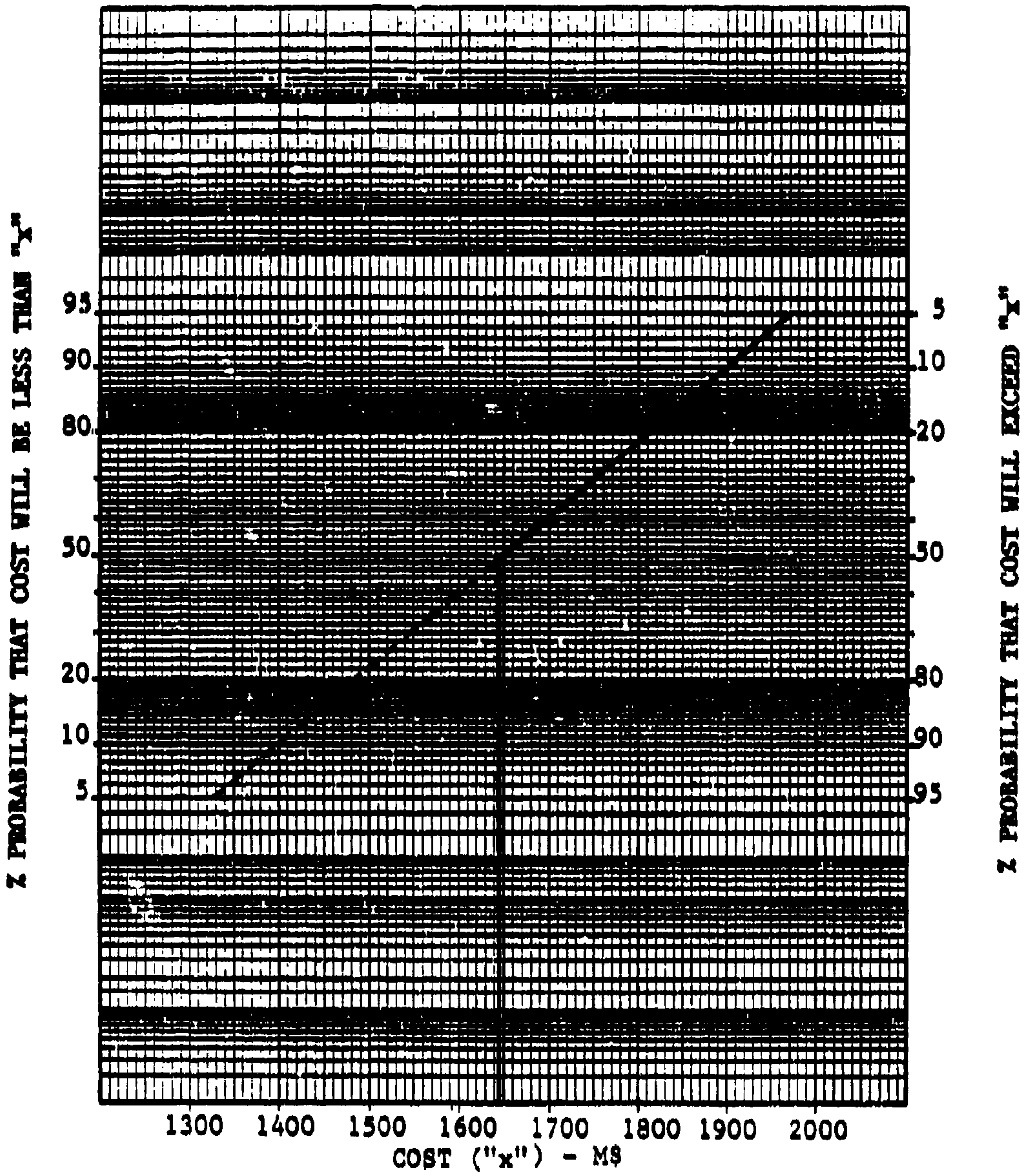

Figure 5. System Cost vs Probability Relationship

C. TRUNCATION RATIONALE

The two basic reasons why system costs are estimated are (1) to aid in determining whether the anticipated benefits warrant the expenditures necessary to acquire the system, and (2) to determine the funding that will be required for its acquisition. A key characteristic of most advanced systems is their relatively large cost - developmental, production, and operations. The funding of a system indicates that the anticipated benefits are judged to be substantially larger than its costs, or else its development would be irrational. The premature loss of the benefits of a key system could have calamitous consequences. Thus, program managers responsible for the acquisition of systems have to balance the dire consequences of diminished performance or premature failure against the expenditures incurred in system acquisition and, as discussed in Sections IV B 1 and IV B 2, would likely opt for enhancing the weakest link(s) rather than underrunning. Consequently, the expected cost (0.5 probability) constitutes a floor on the system cost; therefore, it is recommended that the cost-probability curve be viewed as credible only upward from the expected cost (0.5 probability point).

D. FUNDING LEVEL (RISK-ADJUSTED COST ESTIMATE: R-ACE) DETERMINATION

Although the cost-probability curve is a more valid portrayal of the prevailing reality than a point cost estimate, the budgetary process is incapable of treating a probabilistic spectrum of costs. Therefore, the probabilistic costs depicted on the cost curve need to be translated into a meaningful fixed value for budgetary requirements. A basic guideline for accomplishing this is to use the cost associated with a probability that reflects the level of novelty inherent in the system - the risk-adjusted cost estimate (R-ACE). For example, the cost corresponding to a 0.6 to 0.7 probability level could be used for systems with a substantial legacy from prior systems versus a cost corresponding to a 0.8 to 0.9 probability level for systems incorporating a substantial technological advancement. Major architectural and engineering firms tend to favor a 0.85 probability level for cost risk estimates of new major construction projects.

The source of the cost increase being compensated for by the R-ACE is usually some aspect of actual or potential technological deficiency(s). The presence of a technological deficiency is often referred to as "technical risk." Major categories of technical risks are those that are associated with both the RDT&E phase of a system as well as with its production phase. A prerequisite to the initiation of a successful RDT&E phase (or its cost estimation) is that the physical laws and principles on which the functioning of the system will rely, <u>must</u> be in hand. It is foolhardy to even attempt to estimate the cost of achieving a technical breakthrough. Any cost estimate of a system for which the basic technology is not in hand is meaningless. The history of the nuclear powered aircraft program is brought to mind.

The technical risks in the RDT&E phase are usually associated with the efficient and reliable <u>implementation</u> of known physical laws and principles through engineering design. Even then, substantial risks are occasionally encountered as in the case of the success-eluding program for developing an active refrigerator for cooling spacecraft payload sensors (cryo-cooler). A nominal measure of the impact of the technology implementation risk is recognized as being incorporated within the historically-derived subsystem RDT&E CERs and their standard errors of estimate that constitute the cost estimating data base.

The second area of potential technological deficiencies that create technical risk is associated with production, specifically that stemming from a lack of manufacturing or testing know-how or both. For example, it is one thing to develop and produce a demonstration-of-principle mosaic focal plane containing 16 or even 200 detectors under laboratory conditions; it is quite another to mass produce an operational version with say 10 million detectors. (The "development" of the production facilities may present an even greater technical challenge than the actual development of the detector.)

The resolution of technical risk (i.e., the elimination of technological deficiencies) is accomplished through the acquisition of new knowledge, which may be viewed as a function of time (schedule) and funding (cost). Technical risk per se has no unique intrinsic substance; it manifests itself through some combination of cost and schedule risk - <u>cost</u>, through the focusing of additional skilled scientific and engineering talent on the surmounting or elimination of the impeding deficiencies; schedule through the application of the scientific and engineering talent over a longer period of time or more intensely, which again is basically a cost impact. (Either acceleration or stretch-out of a program from a normalized, baselined schedule usually results in increased costs because of increased labor requirements.) Cost impacts, although they capture the major essence of schedule changes, omit the initial operational capability (IOC) consideration or program criterion. Nevertheless, cost impacts capture most of the essence of schedule risk. Thus, cost impacts constitute an acceptable surrogate for technical risk. This is borne out in the testimony given by Deputy Secretary of Defense, F. C. Carlucci, on 9 February 1982 before the House Committee on Armed Services[13] in which he reported the increases in the FY 83 DOD budget for the technical risks associated with the RDT&E phase of 12 major programs. The average percent funding increase for "technical risk" for the 12 programs was 21.1%. (The median was between 17.7 and 21.1%). Thus, in the absence of other data on which to establish a R-ACE, a cost increase on the order of 20% may be the best estimate possible under the circumstances. However, experience gained from the application of the R-ACE technique would soon provide guidelines for selecting probability levels (and hence costs) commensurate with the inherent uncertainties, nature, and characteristics of the system being costed.

The difference between the cost corresponding to the selected risk probability level and the expected cost should go into a program reserve, possibly such as that recommended in the Total Risk Assessing Cost Estimate (TRACE) concept.[14] Whereas, the budgetary funding request would correspond to the cost associated with the estimated risk probability level, the procurement contract and any incentive/award fees should be based on the proposed, "expected" cost. Were the contract award to be made at say the 0.85 probability cost level, the manifold pressures prevailing on the program management to produce a reliable, high-quality system would assure that the incremental (reserve) funds would be committed to eliminating weak links and enhancing the system, thereby raising the probability of insufficient funds being available to overcome any unexpected adversities late in the program.

The likely net result would be an overrun of the R-ACE amount. To preclude this, the reserve should be in the tight control of the system program office or, in the case of major system contracts, in the control of a top level DOD service board, but not in the control of the contractor, who should endeavor to meet his initial cost estimate.

VI. ADDITIONAL CONSIDERATIONS IN APPLICATION OF METHODOLOGY

A. INFLATION UNCERTAINTY

Although the cost risk methodology presented has described and demonstrated (1) techniques for estimating the cost uncertainty associated with the various cost elements, (2) a recommended methodology for the aggregation of the cost uncertainties, (3) a methodology for the generation of cost-probability curves, and (4) some initial guidelines for the selection of a risk-adjusted cost estimate, consideration of inflation has been excluded to this point for clarity of presentation. All treatment of costs and cost uncertainties has been in base-year dollars. Cost estimates for budgetary requirements generally need to reflect the impacts of inflation and be expressed in then-year dollars. This can be accomplished as described below.

The RDT&E and production costs along with their uncertainties (i.e., standard deviations) can be spread by the use of historically derived spread factors, programmatic schedules, or by other means, over the years in which they are expected to be incurred. To do this, the prorated, or allocated cost estimate as well as its uncertainty estimate for a given year can be generated by multiplying the total RDT&E or production costs and its cost uncertainty σ by the appropriate spread factor. The prorated costs together with the cost uncertainties thus generated for each future year can then be multiplied by the expected inflation factor (e.g., Office of the Secretary of Defense (OSD) inflation factor) for that year. This procedure corresponds to that described under the constant multiplier approach in Section III E 3. Thus spread (allocated) and inflated cost estimates as well as their uncertainties can be generated by direct multiplication by the spread factors and inflation factors.

B. ALTERNATIVE AGGREGATIONS AND COST-PROBABILITY PLOTS

Once the costs and their uncertainty estimates have been spread across the corresponding years and inflated, the final steps are (1) to aggregate the costs and their uncertainties (by adding the standard deviations),

(2) generate cost-probability plots, and (3) select R-ACE values. The direction of aggregation and the resulting plots depend on the cost estimates that are being sought. For example, if annual funding requirements are desired, the aggregation will be by year; if, on the other hand, funding by program phase is desired, aggregation will be by RDT&E phase, production phase, etc.; if total program cost is desired, all costs as well as their associated uncertainties need to be aggregated. After the sought costs, for example annual funding requirements and their uncertainties, are totaled by year, then cost-probability curves can be generated by year and provided to the contracting agency for R-ACE selection of funding requests.

VII. FUTURE EFFORT

In examining areas of deficiencies in cost analysis, two appear to be outstanding and so constitute prime candidates for future effort. The first is the replacement of the method of least squares by the method of least distance for generating cost estimating relationships. When relationships developed by the method of least squares are used to predict costs in the upper ranges of the independent variable, the results are significant underestimates. The method of least distance, which will be described and illustrated in the subsequent effort, does not possess this deficiency.

The second area of deficiency is in the estimation and quantification of key aspects of schedule risk. Results of progress on these two efforts will be reported at the meetings of the Space Systems Cost Analysis Group (SSCAG).

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